$h-$ and $a-$ Formulations for the Time-Domain Modeling of Thin Electromagnetic Shells

R. V. Sabariego*, C. Geuzaine*, P. Dular*◇ and J. Gyselinck†

* Dept. of Electrical Engineering and Computer Science (ACE), University of Liège, ◇FNRS, Belgium

† Dept. of Bio-, Electro- and Mechanical Systems (BEAMS), Université Libre de Bruxelles, Belgium

Abstract

Thin conductive magnetic shells are accounted for by means of two different time-domain magnetodynamic finite element formulations, namely the magnetic field formulation and the magnetic vector potential formulation. Both approaches are an extension of the classical linear frequency-domain thin-shell approximation.

The interface conditions for the magnetic field formulation and the vector potential formulation are expressed in terms of the average current density and the average flux density, respectively, together with a number of higher order components of these quantities.

The proposed time-domain thin-shell approaches are validated by means of a 3-D magnetodynamic problem. The results are shown to agree well with those obtained with a fine model, whereas the computation time is significantly reduced.

1 Introduction

The finite element (FE) analysis of magnetic shielding problems involving thin shells may suffer from both meshing difficulties and high computational cost. These drawbacks can be overcome thanks to the thin-shell approach as the latter allows to reduce the thin-shell volume (thickness d) to an average surface situated halfway between its boundaries. However, it is most often restricted to linear and time-harmonic analyzes [1], [2], [3].

A nonlinear time-domain extension, combining a fixed-point technique and a 1-D FE discretisation of the thin-shell thickness, has been presented in [4]. However, at each fixed-point iteration, the magnetic field inside the shell thickness is computed in the harmonic domain and the residual is then updated in the time domain through an inverse fast Fourier transformation for each geometrical element of the discretisation inside the thin shell.

In [5] the authors proposed a pure time-domain approach with the magnetic vector potential ($a-$)formulation based on the use of even orthogonal polynomial basis functions to account for the variation of the even part of the magnetic flux and electric current density (linked, respectively, to the odd parts of the electric current and magnetic flux density) through the shell thickness. The theoretical developments are presented considering a 3-D formulation and validated through a 2-D application example. The method has been further extended to the nonlinear case in [6].

In this paper, the approach elaborated in [5] is adapted to account for the complete magnetic flux density by means of a set of both even and odd orthogonal polynomial basis functions. The method is further extended to the magnetic field ($h-$)formulation. The performance of both formulations is compared. By way of validation, the thin-shell approach is applied to a 3-D thin magnetic and conducting shield that encloses a coil with a conducting core.

2 Magnetodynamic formulations

Let us consider a magnetodynamic problem in a bounded domain $\Omega = \Omega_c \cup \Omega_c^C \in \mathbb{R}^3$ with boundary $\Gamma = \Gamma_h \cup \Gamma_e$, which is composed of two complementary parts Γ_h and Γ_e (connected or not). The conductive and non-conductive parts of Ω are denoted by Ω_c and Ω_c^C, respectively. Source inductors constitute the domain $\Omega_i \subset \Omega_c^C$ (Figure 1).

The Maxwell equations and constitutive laws governing the low-frequency eddy-current problems are

$$\operatorname{curl} h = j\,, \quad \operatorname{div} b = 0\,, \quad \operatorname{curl} e = -\partial_t\, b\,, \tag{1 a-c}$$

$$b = \mu h\,, \qquad j = \sigma e\,, \tag{1 d e}$$

where h is the magnetic field, b the magnetic flux density (or induction), e the electric field, j the current density, μ the permeability (reluctivity $\nu = 1/\mu$) and σ the conductivity (resistivity $\rho = 1/\sigma$).

2.1 Magnetic vector potential ($a-$)formulation

The $a-$formulation is obtained from the weak form of the Ampere law (1 a):

$$(\nu\,\mathrm{curl}\,a, \mathrm{curl}\,a')_{\Omega} + (\sigma\,\partial_t a, a')_{\Omega_c} + \langle n \times h, a'\rangle_{\Gamma_h} = (j_i, a')_{\Omega_i}\,, \tag{2}$$

where a is the magnetic vector potential; n is the outward unit normal vector on $\Gamma_h \subset \Gamma$; j_i is a prescribed current density; $(\cdot,\cdot)_{\Omega}$ and $\langle\cdot,\cdot\rangle_{\Gamma}$ denote, respectively, a volume integral in Ω and a surface integral on Γ of the scalar product of their two arguments.

2.2 Magnetic field ($\boldsymbol{h}-$)formulation

The counterpart $\boldsymbol{h}-$formulation is obtained from the weak form of the Faraday law (1 c):

$$\partial_t(\mu\, h, h')_{\Omega} + (\rho\,\mathrm{curl}\,h, \mathrm{curl}\,h')_{\Omega_c} + \langle n \times e, h'\rangle_{\Gamma_e} = 0\,, \tag{3}$$

with $\Gamma_e \subset \Gamma$.

The first step in the thin-shell approach consists in reducing the thin-shell volume $\Omega_s \subset \Omega_c$ (thickness d) to an average surface Γ_s situated halfway between the inner surface Γ_s^- and outer surface Γ_s^+ of Ω_s (outward normal n_s), as depicted in Figure 1. The surface integrals in (2) and (3) will then be modified on the basis of a 1-D thin-shell model described hereafter.

3 1-D thin-shell model

In the 1-D model of the shell, only the variation of the magnetic field $h(z,t)$, the flux density $b(z,t)$, the electric field $e(z,t)$ and the current density $j(z,t)$ tangential to the boundary of the shell Γ_s is considered throughout the shell thickness. We adopt a local coordinate system xyz with the $z-$axis normal to the shell (i.e. parallel to n_s) and $z = 0$ at its center, see Figure 2. The tangential components of the magnetic field h and of the electric field e on Γ_s^+ and Γ_s^- (both sides of the shell) are defined as:

$$h_t^+ = n_s \times (h|_{\Gamma_s}^+ \times n_s)\,, \qquad h_t^- = n_s \times (h|_{\Gamma_s}^- \times n_s)\,, \tag{4 a b}$$

$$e_t^+ = n_s \times (e|_{\Gamma_s}^+ \times n_s)\,, \qquad e_t^- = n_s \times (e|_{\Gamma_s}^- \times n_s)\,. \tag{5 a b}$$

Analogously to (4) and (5), hereafter f_t denotes the tangential component of a field f on a surface Γ with normal n.

3.1 $a-$formulation

Governing differential equation. The 1-D eddy-current problem in the shell ($-d/2 \le z \le d/2$) is governed by the following partial differential equation:

$$\partial_z^2 h_t(z,t) = \sigma\, \partial_t b_t(z,t)\,, \tag{6}$$

with constitutive law $h_t(z,t) = \nu\, b_t(z,t)$.

The associated boundary conditions on the upper ($+$) and lower ($-$) surfaces of the shell are given by

$$h_t^+(t) = h_t(d/2,t)\,, \quad h_t^-(t) = h_t(-d/2,t)\,. \tag{7 a b}$$

The average flux density vector $b_0(t)$, tangential to Γ_s, is an essential global quantity. It reads:

$$b_0(t) = \frac{1}{d}\int_{-d/2}^{d/2} b_t(z,t)\, dz\,. \tag{8}$$

Further, taking into account (7) and the Ampere law (1 a), we have:

$$h_t^+ - h_t^- = -n_s \times d\, j_0(t)\,, \tag{9}$$

with $j_0(t)$ the average current density vector (see definition in (19)).

For a sinusoidal time variation at pulsation ω, we define the relative shell thickness as d/δ, with $\delta = \sqrt{2/\sigma\mu\omega}$ the penetration depth. Under these assumptions (6) can be solved analytically, which leads to an expression in terms of the complex representation (symbols in bold) of $h_t^+(t)$, $h_t^-(t)$ and $b_0(t)$ [1]:

$$\boldsymbol{h}_t^+ + \boldsymbol{h}_t^- = 2\,\nu\,\boldsymbol{Y}(d/\delta)\,\boldsymbol{b}_0\,, \tag{10}$$

with

$$\boldsymbol{Y}(d/\delta) = \frac{1+\boldsymbol{\imath}}{2}\,d/\delta\,\text{cotanh}\left(\frac{1+\boldsymbol{\imath}}{2}\,d/\delta\right), \tag{11}$$

where $\boldsymbol{\imath}$ is the imaginary unit. At low frequency, $0 < d/\delta \ll 1$, $\boldsymbol{Y}$ tends towards 1; at sufficiently high frequency, say $d/\delta > 6$, $\boldsymbol{Y}$ is practically equal to $\frac{1+\boldsymbol{\imath}}{2}\,d/\delta$.

The well-known FE frequency-domain approach includes the 1-D thin-shell model in a 2-D and 3-D analysis via the tangential fields h_t^+, h_t^- and (10) as done in [1], [2], [3].

Time-domain extension. We now develop a time-domain extension of (10) by considering $n+1$ polynomial basis functions for the expansion of $b_t(z,t)$. In [5], the authors present a time-domain approximation of (10) based on an expansion of the even part of $b_t(z,t)$ with even orthogonal polynomial basis functions. The odd part of $b_t(z,t)$ is accounted for via the same kind of expansion for the even component of $j_t(z,t)$.

Herein, the complete tangential induction $b_t(z,t)$ is expanded in terms of a set of orthogonal Legendre polynomials $\alpha_k(z)$, i.e. ,

$$b_t(z,t) = \sum\nolimits_{k=0}^{n} \alpha_k(z)\,b_k(t)\,, \tag{12}$$

where $\alpha_0 = 1$, $\alpha_1 = 2\,z/d$, $\alpha_2 = 6\,z^2/d^2 - 1/2$, $\alpha_3 = 20\,z^3/d^3 - 3\,z/d$, $\cdots$ are normalized to verify $|\alpha_k(\pm d/2)| = 1$. Strongly satisfying (6), the magnetic field $h_t(z,t)$ can thus be written as

$$h_t(z,t) = \frac{h_t^+(t) + h_t^-(t)}{2} + \frac{h_t^+(t) - h_t^-(t)}{d}\,z + \sigma d^2 \sum\nolimits_{k=0}^{n} \beta_k(z)\,\partial_t b_k(t)\,, \tag{13}$$

where the polynomials $\beta_k(z)$ verify the following equations: $d^2\,\partial_z^2\,\beta_k(z) = \alpha_k(z)$ and $\beta_k(\pm d/2) = 0$.

Next, when considering a finite number of basis functions, the constitutive law $h(z,t) = \nu\,b(z,t)$, can be weakly imposed as:

$$\int_{-d/2}^{d/2} \alpha_k(z)\,\Big(h_t(z,t) - \nu\,b_t(z,t)\Big)\,\mathrm{d}z = 0\,, \tag{14}$$

which leads to $n+1$ differential equations ($k = 0, \ldots, n$) in terms of $b_0(t), \ldots, b_n(t)$, $h_t^+(t)$ and $h_t^-(t)$.

The following system of linear first-order differential equations is obtained:

$$[H(t)] = \nu\,[P]\,[B(t)] + \sigma d^2\,[Q]\,\partial_t[B(t)]\,, \tag{15}$$

with $[H(t)] = \left[\frac{h_t^+ + h_t^-}{2}\;\frac{h_t^+ - h_t^-}{6}\;0\;\cdots\;0\right]^T$ and $[B(t)] = [b_0(t)\;b_1(t)\;\cdots\;b_n(t)]^T$. The elements p_k and q_{kl} $(k, l = 0, \ldots, n)$ of the diagonal matrix $[P]$ and the symmetric tridiagonal matrix $[Q]$ are given by:

$$p_k = \int_{-d/2}^{d/2} \alpha_k(z)\,\alpha_k(z)\,\mathrm{d}z\,, \quad q_{k,l} = \int_{-d/2}^{d/2} \alpha_k(z)\,\beta_l(z)\,\mathrm{d}z\,. \tag{16}$$

For example, with $n = 1$, these values are $p_0 = 1$, $p_1 = 1/3$, $q_{00} = 1/12$, $q_{01} = q_{10} = 0$ and $q_{11} = 1/60$.

3.2 $h-$formulation

Governing differential equation. Analogously, for the $h-$formulation, the 1-D eddy-current problem in the shell ($-d/2 \leq z \leq d/2$) is governed by:

$$\partial_z^2 e_t(z,t) = \mu\,\partial_t j_t(z,t)\,. \tag{17}$$

with constitutive law $e_t(z,t) = \rho\,j_t(z,t)$.

The associated boundary conditions on the upper $(+)$ and lower $(-)$ surfaces of the shell are given by

$$e_t^+(t) = e_t(d/2,t)\,, \quad e_t^-(t) = e_t(-d/2,t)\,. \tag{18 a b}$$

The relevant global quantity is now the average current density $j_0(t)$, i.e.

$$j_0(t) = \frac{1}{d}\int_{-d/2}^{d/2} j_t(z,t)\,dz\,. \tag{19}$$

Further, taking into account (18) and the Faraday law (1 c), we can write:

$$e_t^+ - e_t^- = n_s \times d\,\partial_t b_0(t)\,. \tag{20}$$

In the linear harmonic case, analogously to the case of the $a-$formulation, the analytical solution of (17) can be written in terms of the complex representation of $e_t^+(t)$, $e_t^-(t)$ and $j_0(t)$ [1]:

$$\boldsymbol{e}_t^+ + \boldsymbol{e}_t^- = 2\,\rho\,\boldsymbol{Y}(d/\delta)\,\boldsymbol{j}_0\,. \tag{21}$$

Time-domain extension. We consider $n+1$ polynomial basis functions for the expansion of $j_t(z,t)$ to develop a time-domain extension of (21). The tangential current density $j_t(z,t)$ is thus expanded in terms of the Legendre polynomials $\alpha_k(z)$, i.e. ,

$$j_t(z,t) = \sum\nolimits_{k=0}^{n} \alpha_k(z)\, j_k(t)\,. \tag{22}$$

Strongly satisfying (17), the magnetic field $e_t(z,t)$ can thus be written as

$$e_t(z,t) = \frac{e_t^+(t) + e_t^-(t)}{2} + \frac{e_t^+(t) - e_t^-(t)}{d}\, z + \mu d^2 \sum\nolimits_{k=0}^{n} \beta_k(z)\, \partial_t e_k(t)\,. \tag{23}$$

We consider a finite number of basis functions and weakly impose the constitutive law $e_t(z,t) = \rho\, j_t(z,t)$ as:

$$\int_{-d/2}^{d/2} \alpha_k(z) \left(e_t(z,t) - \rho\, j_t(z,t) \right) \mathrm{d}z = 0\,, \tag{24}$$

which leads to $n+1$ differential equations ($k = 0, \ldots, n$) in terms of $j_0(t), \ldots, j_n(t)$, $e_t^+(t)$ and $e_t^-(t)$. The following system of linear first-order differential equations is obtained:

$$[E(t)] = \rho\, [P]\, [J(t)] + \mu d^2\, [Q]\, \partial_t [J(t)]\,, \tag{25}$$

with $[E(t)] = \left[\frac{e_t^+ + e_t^-}{2} \;\; \frac{e_t^+ - e_t^-}{6} \;\; 0 \; \cdots \; 0 \right]^T$ and $[J(t)] = [j_0(t)\; j_1(t) \;\; \cdots \;\; j_n(t)]^T$. The matrices $[P]$ and $[Q]$ are the same as in (15).

4 FE implementation

In the thin-shell formulation, the thin-shell volume Ω_s is excluded from the original calculation domain Ω. Further, the surface Γ_s with outward normal n_s and situated halfway between the inner surface Γ_s^- and outer surface Γ_s^+ of Ω_s is added to the new domain $\Omega \backslash \Omega_s$ (Figure 1). In order to account for the changes in these domains, the surface integral term in (2) and (3) are modified. We consider that surfaces Γ_s^- and Γ_s^+ have slightly moved so that they coincide with the average surface Γ_s. For the sake of simplicity, we abuse notation and keep on referring to them with the same symbols.

4.1 $a-$formulation

The new weak form reads:

$$(\nu\,\mathrm{curl}\, a, \mathrm{curl}\, a')_{\Omega\backslash\Omega_s} + (\sigma\,\partial_t a, a')_{\Omega_c} + \langle n \times h, a' \rangle_{\Gamma_h} + \langle n_s \times h, a' \rangle_{\Gamma_s^-} - \langle n_s \times h, a' \rangle_{\Gamma_s^+} = (j_i, a')_{\Omega_i}\,. \quad (26)$$

The time-domain behavior of the thin shell is taken into account by introducing the tangential vector fields $b_0,\, b_1,\, \cdots\, b_n$ on Γ_s as unknowns.

The tangential component of the magnetic vector potential a_t is discontinuous across Γ_s and is related to the net flux $d\,b_0$ in the shell as

$$a_t^+ - a_t^- = -n_s \times d\,b_0(t)\,. \quad (27)$$

We therefore decompose a as $a_c + a_d$, the tangential components of a_c and a_d being continuous and discontinuous across the shell, respectively [3].

By considering $a^- = a_c$, $a^+ = a_c + a_d$ and $a_d = -n_s \times d\,b_0$ together with (9), we can work out the two new surface terms in (26). We thus obtain

$$\begin{aligned}\langle n_s \times h, a' \rangle_{\Gamma_s^-} - \langle n_s \times h, a' \rangle_{\Gamma_s^+} &= -\langle n_s \times h_t^+, a_c' \rangle_{\Gamma_s} - \langle n_s \times h_t^+, a_d' \rangle_{\Gamma_s} + \langle n_s \times h_t^-, a_c' \rangle_{\Gamma_s} \\ &= d\langle h_t^+, b_0' \rangle_{\Gamma_s} - d\langle j_0, a_c' \rangle_{\Gamma_s}\,, \end{aligned} \quad (28)$$

where j_0 is obtained from (1 a) and (13).

Using the first two equations of system (15) we get an expression for h_t^+ and h_t^- in terms of b_0, b_1, b_2 and b_3 (assuming $n \geq 2$), i.e.

$$h_t^{\pm} = \nu\, b_0 + \sigma d^2 (q_{00}\partial_t b_0 + q_{02}\partial_t b_2) \pm 3\,\nu\, b_1 \pm 3\,\sigma d^2 (q_{01}\partial_t b_1 + q_{03}\partial_t b_3)\,, \quad (29)$$

where the upper (lower) superscript corresponds to the upper (lower) sign. The weak form (26) is thus coupled with the time-domain thin-shell approximation via a_c, a_d in $\Omega\backslash\Omega_s$ and b_0, b_1, b_2 and b_3 on Γ_s.

Next, from (9) and applying the Ampere law (1 a) to (13), we get the second condition concerning the tangential components of a_c and a_d. We have:

$$-\frac{1}{2}\sigma\partial_t(2a_{c,t} + a_{d,t}) = \frac{2}{d}\,\nu b_1 + \sigma d\Big(\frac{1}{5}\,\partial_t b_1 - \frac{1}{70}\,\partial_t b_3\Big)\,, \quad (30)$$

which we can weakly impose on Γ_s with test functions b'_1 and b'_3.

The remaining equations of system (15) result in the following weak forms with test functions b'_l ($l = 2, 3, \cdots, n$):

$$0 = \langle \nu\, p_l\, b_l, b'_l \rangle_{\Gamma_s} + \sum_{i=-2,0,2} \langle \sigma d^2\, q_{l,l+i}\, \partial_t b_{l+i}, b'_l \rangle_{\Gamma_s} \,. \tag{31}$$

4.2 $h-$formulation

The weak form of the Faraday law is modified as:

$$\partial_t(\mu\, h, h')_{\Omega} + (\rho\, \mathrm{curl}\, h, \mathrm{curl}\, h')_{\Omega_c} + \langle n \times e, h' \rangle_{\Gamma_e} + \langle n_s \times e, h' \rangle_{\Gamma_s^-} - \langle n_s \times e, h' \rangle_{\Gamma_s^+} = 0\,, \tag{32}$$

From (9), the net current $d\, j_0$ in the shell implies that the tangential component of the magnetic field is discontinuous across the shell. We thus decompose h as $h_c + h_d$, the tangential components of which are continuous and discontinuous across Ω_s, respectively.

By considering $h^- = h_c$ and $h^+ = h_c - n_s \times d\, j_0$, and taking into account (18) and (20), we work out the two new surface terms in (32):

$$\begin{aligned}\langle n_s \times e, h' \rangle_{\Gamma_s^-} - \langle n_s \times e, h' \rangle_{\Gamma_s^+} &= -\langle n_s \times e_t^+, h'_c \rangle_{\Gamma_s} - \langle n_s \times e_t^+, h'_d \rangle_{\Gamma_s} + \langle n_s \times e_t^-, h'_c \rangle_{\Gamma_s} \\ &= d\, \langle e_t^+, j_0 \rangle_{\Gamma_s} - d\, \langle \partial_t b_0, h'_c \rangle_{\Gamma_s} \,.\end{aligned} \tag{33}$$

Using the first two lines of system (25) we get an expression for e_t^+ and e_t^- in terms of j_0, j_1, j_2 and j_3 (assuming $n \geq 2$), i.e.

$$e_t^{\pm} = \rho\, j_0 + \mu d^2 (q_{00} \partial_t j_0 + q_{02} \partial_t j_2) \pm 3\, \rho\, j_1 \pm 3\, \mu d^2 (q_{01} \partial_t j_1 + q_{03} \partial_t j_3)\,. \tag{34}$$

The weak form (32) is thus coupled with the time-domain thin-shell approximation via h_c, h_d in $\Omega \backslash \Omega_s$ and j_0, j_1, j_2 and j_3 on Γ_s.

The second condition concerning the tangential components $h_{c,t}$ and $h_{d,t}$ is obtained from (20) and applying the Faraday law (1 c) to (23). We have:

$$-\frac{1}{2} \mu \partial_t (2 h_{c,t} + h_{d,t}) = \frac{2}{d}\, \rho j_1 + \mu d \Big(\frac{1}{5}\, \partial_t j_1 - \frac{1}{70}\, \partial_t j_3 \Big)\,, \tag{35}$$

which we can weakly impose on Γ_s with test functions j'_1 and j'_3.

The remaining equations of system (25) complete the formulation with test functions j'_l ($l = 2, 3, \cdots, n$):

$$0 = \langle \rho\, p_l\, j_l, j'_l \rangle_{\Gamma_s} + \sum_{i=-2,0,2} \langle \mu d^2\, q_{l,l+i}\, \partial_t j_{l+i}, j'_l \rangle_{\Gamma_s} \,. \tag{36}$$

5 Application example

We consider a magnetic shield ($\mu_r = 1000$, $\sigma = 2\,10^6$ S/m) enclosing a coil with a conducting core ($\mu_r = 1$, $\sigma = 3.7\,10^7$ S/m). Taking advantage of the symmetry, only 1/16th of the structure is modeled, see Figure 3 [7].

The thin-shell approach is applied taking a number of terms $n = 0, 2, 4$ for the expansion of either b_t with the $a-$formulation or j_t with the $h-$formulation. A reference model with a fine discretization of the thin-shell volume (number of layers of elements equal to $4\,\max(d/\delta, 1)$) provides an accurate solution. Global results, viz the total joule losses and magnetic energy, in the plate are compared.

Time-stepping simulations with imposed sinusoidal current of fundamental frequency $f = 1.24\,\text{kHz}$ ($d/\delta = 5$) and amplitude 3000 At are carried out. One period $T = 1/f = 0.808\,\text{ms}$ is time-stepped with $\Delta t = T/120$.

The joule losses in the shield as a function of time, obtained with the classical formulations and the thin-shell approximations, are shown in Figure 4. The relative error for the joule losses of the thin-shell approximations is depicted in Figure 5. The magnetic energy in the shield as a function of time is represented in Figure 6. The relative error of the thin-shell approximation is shown in Figure 7 as well. For both considered global quantities and with both the $a-$ and $h-$formulations, the error clearly decreases with increasing n and is of the same order of magnitude.

For the frequency at hand, an excellent agreement is observed between the reference models and the corresponding thin-shell approximations for $n = 4$, i.e. by introducing b_0, b_1, b_2, b_3 and b_4 with the $a-$formulation or j_0, j_1, j_2, j_3 and j_4 with the $h-$formulation.

5.1 Computational cost

In order to highlight the effectiveness of the presented time-domain thin-shell approaches, we analyze the computational cost. The systems of algebraic equations are solved by means of the iterative solver GMRES [8] with ILU-preconditioning on a 2.26 GHz Intel Pentium M Processor.

In the reference model, the shield is discretized with 20 layers of elements of the same width ($d/\delta = 5$) what yields N real unknowns associated to the edges of the complete mesh with the conventional $a-$formulation and $h-$formulation (see Table I). With the thin-shell approach and $n = 4$, the number of unknowns N is reduced by a factor 2 with both formulations. The computation time is roughly reduced by a factor 4.6 and 6.3 with the $a-$ and $h-$formulations, respectively. See Table I for further details. A significant speed-up is thus achieved.

For a higher ratio d/δ, the number of unknowns of the reference models increases much faster than the value of n of additional unknowns required for ensuring a prescribed accuracy. Indeed, note that with the proposed thin-shell approaches, increasing n with 2 units implies an increment of only 2726 unknowns in both approximations. Therefore, for reaching a value N comparable to those of the fine models, n should be higher than 16, while an excellent agreement is already observed for $n = 4$. The efficiency of the method in terms of memory requirements and computational cost is thus clear.

6 Conclusions

Two counterpart thin-shell time-domain finite-element formulations have been elaborated based on the conventional magnetic vector potential and magnetic field formulations. The proposed methods are based on the coupling of a time-domain 1-D thin-shell model with the surface-integral term in either the magnetic vector potential formulation or the magnetic field formulation. With that purpose, a number of additional unknowns for the current density or the flux density are associated to the shell boundary.

A clear advantage of the proposed thin-shell approach is that the mesh of the shell surface does not depend on the frequency, unlike the mesh of the thin-shell volume in the conventional approach. Furthermore, for a given accuracy, when increasing the frequency, the number of additional unknowns is very limited in comparison with those required by the classical model. The method allows for a good compromise between computational cost and accuracy. Indeed, adding a sufficiently high number of either induction components ($a-$formulation) or current density components ($h-$formulation) in the thin-shell, a sufficiently high precision can be achieved.

Acknowledgement

The research was supported by the Belgian Science Policy (IAP P6/21).

References

[1] L. Krähenbühl and D. Muller, "Thin layers in electrical engineering. example of shell models in analyzing eddy-currents by boundary and finite element methods," *IEEE Trans. on Magn.*, vol. 29, no. 5, pp. 1450–1455, 1993.

[2] I. D. Mayergoyz and G. Bedrosian, "On calculation of 3-D eddy currents in conducting and magnetic shells," *IEEE Trans. on Magn.*, vol. 31, no. 3, pp. 1319–1324, 1995.

[3] C. Geuzaine, P. Dular, and W. Legros, "Dual formulations for the modeling of thin electromagnetic shells using edge elements," *IEEE Trans. on Magn.*, vol. 36, no. 4, pp. 799–803, 2000.

[4] O. Bottauscio, M. Chiampi, and A. Manzin, "Transient analysis of thin layers for the magnetic field shielding," *IEEE Trans. on Magn.*, vol. 42, no. 4, pp. 871–874, 2006.

[5] J. Gyselinck, R. V. Sabariego, P. Dular, and C. Geuzaine, "Time-domain finite-element modelling of thin electromagnetic shells," *IEEE Trans. on Magn.*, vol. 44, no. 6, pp. 742–745, 2008.

[6] R. V. Sabariego, C. Geuzaine, P. Dular, and J. Gyselinck, "Nonlinear time-domain finite-element modeling of thin electromagnetic shells," in *Proceedings of the 13th Biennial IEEE Conference on Electromagnetic Field Computation (CEFC2008)*, Athens, Greece, May 11–15, 2008.

[7] T. Nakata, N. Takahashi, K. Fujiwara, and T. Imai, "Effects of permeability of magnetic materials on errors of the T-ω method," *IEEE Trans. on Magn.*, vol. 26, no. 2, pp. 698–701, 1990.

[8] Y. Saad and M. H. Shultz, “GMRES: a generalized minimal residual algorithm for solving nonsymmetric linear systems,” *SIAM Journal on Scientific Computing*, vol. 7, no. 3, pp. 856–869, 1986.

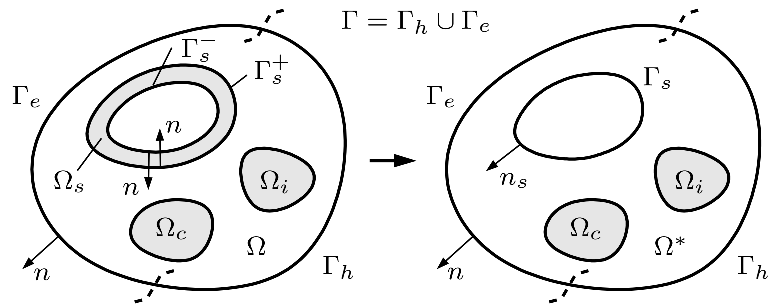

Fig. 1 Calculation domain Ω and reduction of the thin-shell domain Ω_s to the surface Γ_s

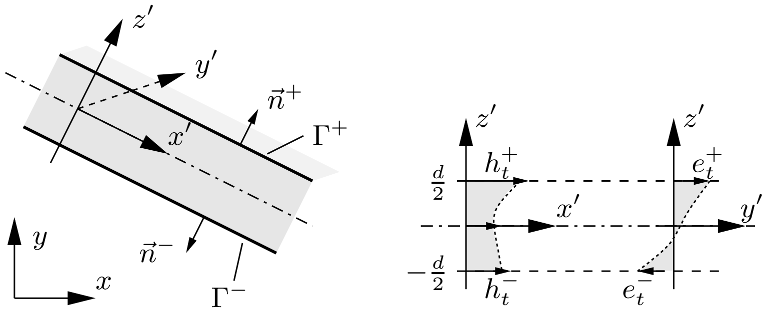

Fig. 2 Local coordinate system xyz for the 1-D thin-shell model

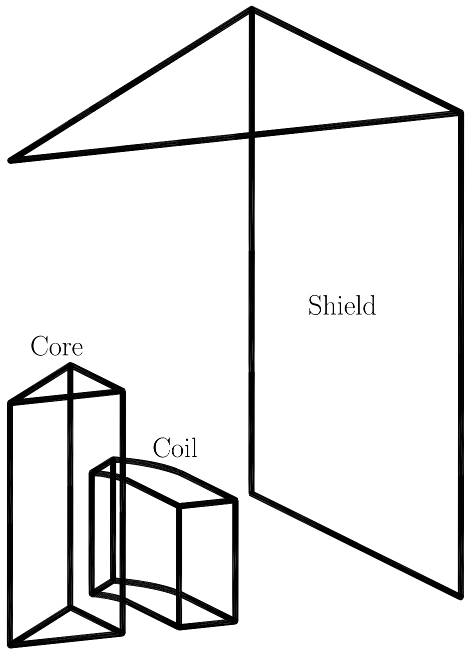

Fig. 3 Magnetic shield enclosing a coil with a conducting core, 1/16th of the structure

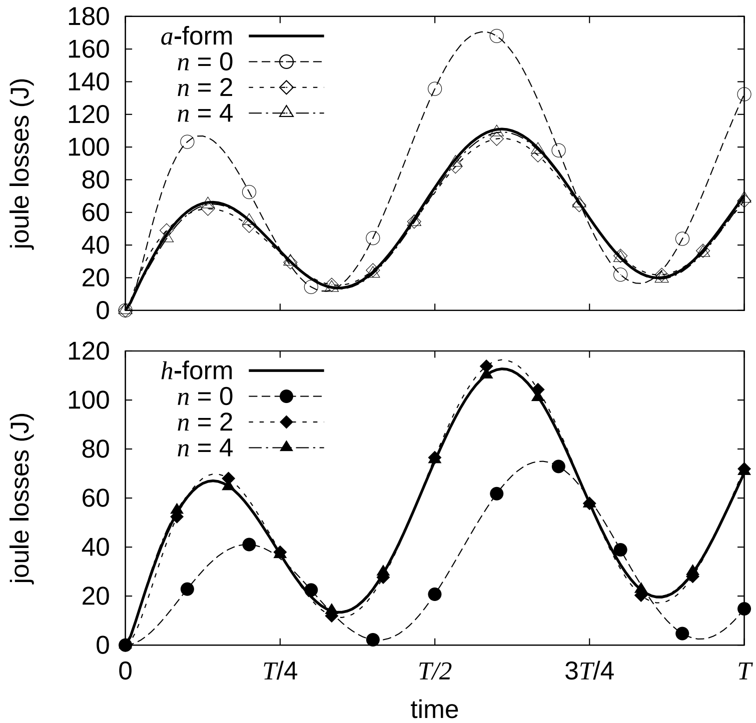

Fig. 4 Joule losses in the thin shell with imposed sinusoidal current ($d/\delta = 5$). Comparison between reference model and the associated thin-shell models for $n = 0, 2, 4$. $a-$formulation (up), $h-$formulation (down)

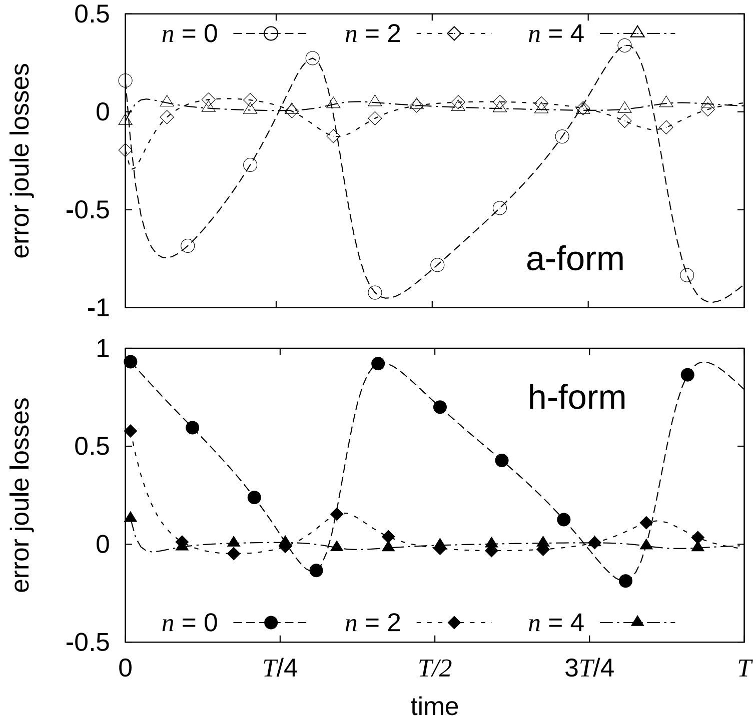

Fig. 5 Relative error of the joule losses in the thin shell with regard to the reference model for $n = 0, 2, 4$. $a-$formulation (up), $h-$formulation (down)

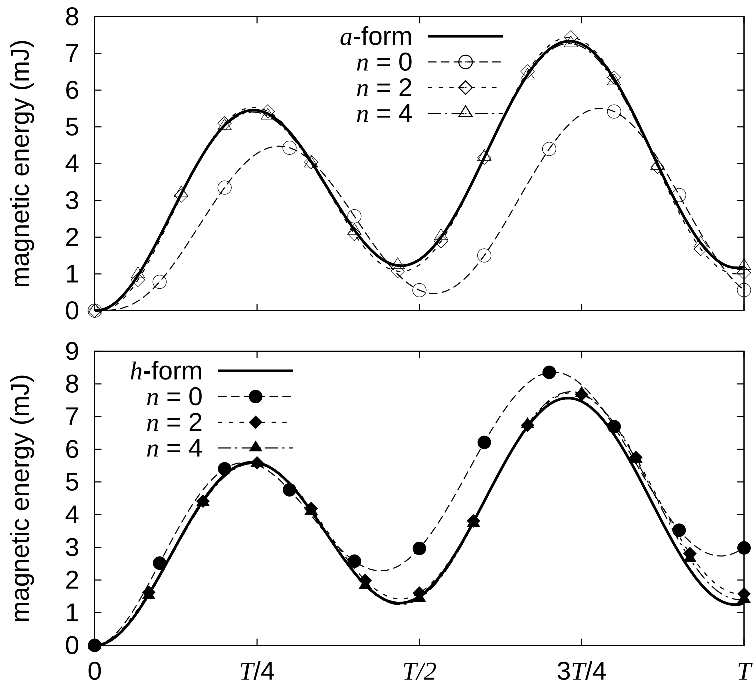

Fig. 6 Magnetic energy in the thin shell with imposed sinusoidal current ($d/\delta = 5$). Comparison between reference model and the associated thin-shell models for $n = 0, 2, 4$. $a-$formulation (up), $h-$formulation (down)

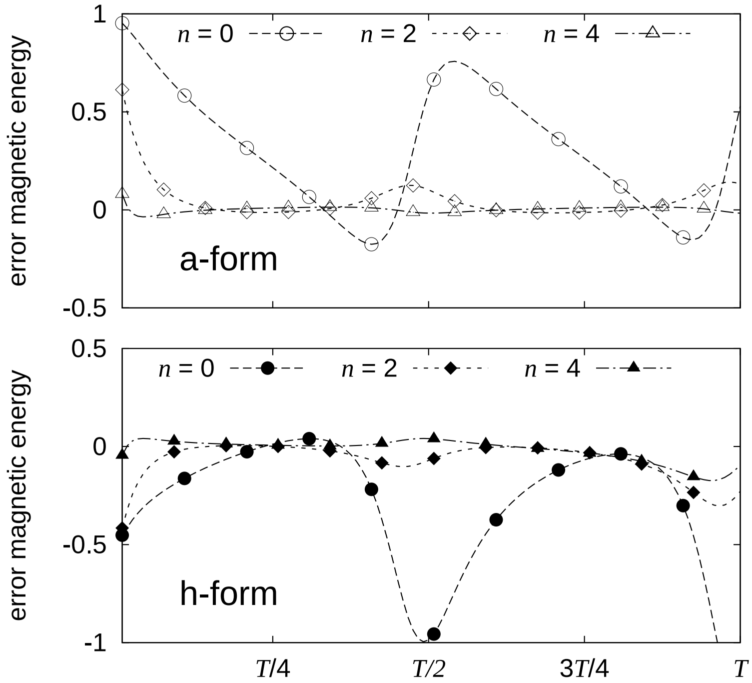

Fig. 7 Relative error of the magnetic energy in the thin shell with regard to the reference model for $n = 0, 2, 4$. $a-$formulation (up), $h-$formulation (down)

TABLE I - Computation time t and number of degrees of freedom N for conventional FE formulations and thin-shell approaches

	$a-$formulation		$h-$formulation	
model	N	t (s)	N	t (s)
fine	83108	158.4	74244	82.2
$n = 0$	38692	30.7	25785	13.1
$n = 2$	41418	32.4	28511	13.5
$n = 4$	44144	34.3	31237	13.6